Further Theorems of Propositional Calculus

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Abstract

This module includes proofs of popositional calculus theorems. The following theorems and proofs are adapted from D. Hilbert and W. Ackermann's 'Grundzuege der theoretischen Logik' (Berlin 1928, Springer)

Specification

This document has the following specification:

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References

This document uses the results of the following documents:

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pdf:	prophilbert2_1.00.00_1.02.00.pdf

Content

First distributive law (first direction):

Theorem 0.1 (hilb36).

$$(P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))$$

Proof.

1	$(P \land Q) \to P$	add sentence hilb24
2	$(P \land A) \to P$	replace Q by A in 1
3	$(B \land A) \to B$	replace P by B in 2
4	$(Q \land A) \to Q$	replace B by Q in 3
5	$(P \to Q) \to ((A \lor P) \to (A \lor Q))$	add axiom axiom4
6	$(P \to Q) \to ((B \lor P) \to (B \lor Q))$	replace A by B in 5
7	$(P \to C) \to ((B \lor P) \to (B \lor C))$	replace Q by C in 6
8	$(D \to C) \to ((B \lor D) \to (B \lor C))$	replace P by D in 7
9	$(D \to C) \to ((P \lor D) \to (P \lor C))$	replace B by P in 8
10	$(D \to Q) \to ((P \lor D) \to (P \lor Q))$	replace C by Q in 9
11	$((Q \land A) \to Q) \to ((P \lor (Q \land A)) \to (P \lor Q))$	replace D by $Q \land A$ in 10
12	$(P \lor (Q \land A)) \to (P \lor Q)$	MP with 4, 11
13	$(P \land Q) \to Q$	add sentence hilb25
14	$(P \land A) \to A$	replace Q by A in 13
15	$(B \land A) \to A$	replace P by B in 14
16	$(Q \land A) \to A$	replace B by Q in 15
17	$(D \to A) \to ((P \lor D) \to (P \lor A))$	replace C by A in 9
18	$((Q \land A) \to A) \to ((P \lor (Q \land A)) \to (P \lor A))$	replace D by $Q \land A$ in 17
19	$(P \lor (Q \land A)) \to (P \lor A)$	MP with 16, 18
20	$P \to (Q \to (P \land Q))$	add sentence hilb28
21	$P \to (A \to (P \land A))$	replace Q by A in 20
22	$B \to (A \to (B \land A))$	replace P by B in 21
23	$B \to ((P \lor A) \to (B \land (P \lor A)))$	replace A by $P \lor A$ in 22
24	$(P \lor Q) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))$	replace B by $P \lor Q$ in 23
25	$(P \to Q) \to ((A \to P) \to (A \to Q))$	add sentence hilb1
26	$(P \to Q) \to ((B \to P) \to (B \to Q))$	replace A by B in 25
27	$(P \to C) \to ((B \to P) \to (B \to C))$	replace Q by C in 26
28	$(D \to C) \to ((B \to D) \to (B \to C))$	replace P by D in 27
29	$(D \to C) \to (((P \lor (Q \land A)) \to D) \to ((P \lor (Q \land A)) \to C))$	replace B by $P \lor (Q \land A)$ in 28
30	$(D \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))) \to (((P \lor (Q \land A)) \to D) \to ((P \lor (Q \land A)) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))))$	replace C by $(P \lor A) \to ((P \lor Q) \land (P \lor A))$ in 29
31	$((P \lor Q) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))) \to (((P \lor (Q \land A)) \to (P \lor Q)) \to ((P \lor (Q \land A)) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))))$	replace D by $P \lor Q$ in 30
32	$((P \lor (Q \land A)) \to (P \lor Q)) \to ((P \lor (Q \land A)) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A))))$	MP with 24, 31
33	$(P \lor (Q \land A)) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))$	MP with 12, 32
34	$(P \to (Q \to A)) \to (Q \to (P \to A))$	add sentence hilb16
35	$(P \to (Q \to B)) \to (Q \to (P \to B))$	replace A by B in 34
36	$(P \to (C \to B)) \to (C \to (P \to B))$	replace Q by C in 35
37	$(D \to (C \to B)) \to (C \to (D \to B))$	replace P by D in 36
38	$(D \to (C \to ((P \lor Q) \land (P \lor A)))) \to (C \to (D \to ((P \lor Q) \land (P \lor A))))$	replace B by $(P \lor Q) \land (P \lor A)$ in 37
39	$(D \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))) \to ((P \lor A) \to (D \to ((P \lor Q) \land (P \lor A))))$	replace C by $P \lor A$ in 38

40	$((P \lor (Q \land A)) \to ((P \lor A) \to ((P \lor Q) \land (P \lor A)))) \to ((P \lor A) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))))$	replace D by $P \lor (Q \land A)$ in 39
41	$(P \lor A) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))$	MP with 33, 40
42	$(D \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))) \to (((P \lor (Q \land A)) \to D) \to ((P \lor (Q \land A)) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))))$	replace C by $(P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))$ in 29
43	$((P \lor A) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))) \to (((P \lor (Q \land A)) \to (P \lor A)) \to ((P \lor (Q \land A)) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))))$	replace D by $P \lor A$ in 42
44	$((P \lor (Q \land A)) \to (P \lor A)) \to ((P \lor (Q \land A)) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))))$	MP with 41, 43
45	$(P \lor (Q \land A)) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))$	MP with 19, 44
46	$(P \to (P \to Q)) \to (P \to Q)$	add sentence hilb33
47	$(P \to (P \to A)) \to (P \to A)$	replace Q by A in 46
48	$(B \to (B \to A)) \to (B \to A)$	replace P by B in 47
49	$(B \to (B \to ((P \lor Q) \land (P \lor A)))) \to (B \to ((P \lor Q) \land (P \lor A)))$	replace A by $(P \lor Q) \land (P \lor A)$ in 48
50	$((P \lor (Q \land A)) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))) \to ((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A)))$	replace B by $P \lor (Q \land A)$ in 49
51	$(P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))$	MP with 45, 50

□

First distributive law (second direction):

Theorem 0.2 (hilb37).

$$((P \lor Q) \land (P \lor A)) \to (P \lor (Q \land A))$$

Proof.

1	$P \to (Q \to (P \land Q))$	add sentence hilb28
2	$P \to (A \to (P \land A))$	replace Q by A in 1
3	$B \to (A \to (B \land A))$	replace P by B in 2
4	$Q \to (A \to (Q \land A))$	replace B by Q in 3
5	$(P \to Q) \to ((A \lor P) \to (A \lor Q))$	add axiom axiom4
6	$(P \to Q) \to ((B \lor P) \to (B \lor Q))$	replace A by B in 5
7	$(P \to C) \to ((B \lor P) \to (B \lor C))$	replace Q by C in 6
8	$(D \to C) \to ((B \lor D) \to (B \lor C))$	replace P by D in 7
9	$(D \to C) \to ((P \lor D) \to (P \lor C))$	replace B by P in 8
10	$(D \to (Q \land A)) \to ((P \lor D) \to (P \lor (Q \land A)))$	replace C by $Q \land A$ in 9
11	$(A \to (Q \land A)) \to ((P \lor A) \to (P \lor (Q \land A)))$	replace D by A in 10
12	$(P \to Q) \to ((A \to P) \to (A \to Q))$	add sentence hilb1
13	$(P \to Q) \to ((B \to P) \to (B \to Q))$	replace A by B in 12
14	$(P \to C) \to ((B \to P) \to (B \to C))$	replace Q by C in 13
15	$(D \to C) \to ((B \to D) \to (B \to C))$	replace P by D in 14
16	$(D \to C) \to ((Q \to D) \to (Q \to C))$	replace B by Q in 15
17	$(D \to ((P \lor A) \to (P \lor (Q \land A)))) \to ((Q \to D) \to (Q \to ((P \lor A) \to (P \lor (Q \land A)))))$	replace C by $(P \lor A) \to (P \lor (Q \land A))$ in 16
18	$((A \to (Q \land A)) \to ((P \lor A) \to (P \lor (Q \land A)))) \to ((Q \to (A \to (Q \land A))) \to (Q \to ((P \lor A) \to (P \lor (Q \land A)))))$	replace D by $A \to (Q \land A)$ in 17

19	$(Q \to (A \to (Q \wedge A))) \to (Q \to ((P \vee A) \to (P \vee (Q \wedge A))))$	MP with 11, 18
20	$Q \to ((P \vee A) \to (P \vee (Q \wedge A)))$	MP with 4, 19
21	$(P \to (Q \to A)) \to (Q \to (P \to A))$	add sentence hilb16
22	$(P \to (Q \to B)) \to (Q \to (P \to B))$	replace A by B in 21
23	$(P \to (C \to B)) \to (C \to (P \to B))$	replace Q by C in 22
24	$(D \to (C \to B)) \to (C \to (D \to B))$	replace P by D in 23
25	$(D \to (C \to (P \vee (Q \wedge A)))) \to (C \to (D \to (P \vee (Q \wedge A))))$	replace B by $P \vee (Q \wedge A)$ in 24
26	$(D \to ((P \vee A) \to (P \vee (Q \wedge A)))) \to ((P \vee A) \to (D \to (P \vee (Q \wedge A))))$	replace C by $P \vee A$ in 25
27	$(Q \to ((P \vee A) \to (P \vee (Q \wedge A)))) \to ((P \vee A) \to (Q \to (P \vee (Q \wedge A))))$	replace D by Q in 26
28	$(P \vee A) \to (Q \to (P \vee (Q \wedge A)))$	MP with 20, 27
29	$(D \to (P \vee (Q \wedge A))) \to ((P \vee D) \to (P \vee (P \vee (Q \wedge A))))$	replace C by $P \vee (Q \wedge A)$ in 9
30	$(Q \to (P \vee (Q \wedge A))) \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A))))$	replace D by Q in 29
31	$(D \to C) \to (((P \vee A) \to D) \to ((P \vee A) \to C))$	replace B by $P \vee A$ in 15
32	$(D \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A))))) \to (((P \vee A) \to D) \to ((P \vee A) \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A))))))$	replace C by $(P \vee Q) \to (P \vee (P \vee (Q \wedge A)))$ in 31
33	$((Q \to (P \vee (Q \wedge A))) \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A))))) \to (((P \vee A) \to (Q \to (P \vee (Q \wedge A)))) \to ((P \vee A) \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A))))))$	replace D by $Q \to (P \vee (Q \wedge A))$ in 32
34	$((P \vee A) \to (Q \to (P \vee (Q \wedge A)))) \to ((P \vee A) \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A)))))$	MP with 30, 33
35	$(P \vee A) \to ((P \vee Q) \to (P \vee (P \vee (Q \wedge A))))$	MP with 28, 34
36	$(P \vee A) \to ((P \vee Q) \to ((P \vee P) \vee (Q \wedge A)))$	elementary equivalence in 35 at 1 of hilb14 with hilb14
37	$(P \vee A) \to ((P \vee Q) \to (P \vee (Q \wedge A)))$	elementary equivalence in 36 at 1 of hilb11 with hilb11
38	$(D \to ((P \vee Q) \to (P \vee (Q \wedge A)))) \to ((P \vee Q) \to (D \to (P \vee (Q \wedge A))))$	replace C by $P \vee Q$ in 25
39	$((P \vee A) \to ((P \vee Q) \to (P \vee (Q \wedge A)))) \to ((P \vee Q) \to ((P \vee A) \to (P \vee (Q \wedge A))))$	replace D by $P \vee A$ in 38
40	$(P \vee Q) \to ((P \vee A) \to (P \vee (Q \wedge A)))$	MP with 37, 39
41	$(P \to (Q \to A)) \to ((P \wedge Q) \to A)$	add sentence hilb29
42	$(P \to (Q \to B)) \to ((P \wedge Q) \to B)$	replace A by B in 41
43	$(P \to (C \to B)) \to ((P \wedge C) \to B)$	replace Q by C in 42
44	$(D \to (C \to B)) \to ((D \wedge C) \to B)$	replace P by D in 43
45	$(D \to (C \to (P \vee (Q \wedge A)))) \to ((D \wedge C) \to (P \vee (Q \wedge A)))$	replace B by $P \vee (Q \wedge A)$ in 44
46	$(D \to ((P \vee A) \to (P \vee (Q \wedge A)))) \to ((D \wedge (P \vee A)) \to (P \vee (Q \wedge A)))$	replace C by $P \vee A$ in 45
47	$((P \vee Q) \to ((P \vee A) \to (P \vee (Q \wedge A)))) \to (((P \vee Q) \wedge (P \vee A)) \to (P \vee (Q \wedge A)))$	replace D by $P \vee Q$ in 46
48	$((P \vee Q) \wedge (P \vee A)) \to (P \vee (Q \wedge A))$	MP with 40, 47

□

A form for the abbreviation rule form for disjunction (first direction):

Theorem 0.3 (hilb38).

$$(P \lor Q) \to \neg(\neg P \land \neg Q)$$

Proof.

1	$P \to P$	add sentence hilb2
2	$Q \to Q$	replace P by Q in 1
3	$(P \lor Q) \to (P \lor Q)$	replace Q by $P \lor Q$ in 2
4	$(P \lor Q) \to \neg\neg(P \lor Q)$	elementary equivalence in 3 at 5 of hilb5 with hilb5
5	$(P \lor Q) \to \neg\neg(\neg\neg P \lor Q)$	elementary equivalence in 4 at 8 of hilb5 with hilb5
6	$(P \lor Q) \to \neg\neg(\neg\neg P \lor \neg\neg Q)$	elementary equivalence in 5 at 11 of hilb5 with hilb5
7	$(P \lor Q) \to \neg(\neg P \land \neg Q)$	reverse abbreviation and in 6 at occurence 1

□

A form for the abbreviation rule form for disjunction (second direction):

Theorem 0.4 (hilb39).

$$\neg(\neg P \land \neg Q) \to (P \lor Q)$$

Proof.

1	$P \to P$	add sentence hilb2
2	$Q \to Q$	replace P by Q in 1
3	$(P \lor Q) \to (P \lor Q)$	replace Q by $P \lor Q$ in 2
4	$\neg\neg(P \lor Q) \to (P \lor Q)$	elementary equivalence in 3 at 2 of hilb5 with hilb5
5	$\neg\neg(\neg\neg P \lor Q) \to (P \lor Q)$	elementary equivalence in 4 at 5 of hilb5 with hilb5
6	$\neg\neg(\neg\neg P \lor \neg\neg Q) \to (P \lor Q)$	elementary equivalence in 5 at 8 of hilb5 with hilb5
7	$\neg(\neg P \land \neg Q) \to (P \lor Q)$	reverse abbreviation and in 6 at occurence 1

□

By duality we get the second distributive law (first direction):

Theorem 0.5 (hilb40).

$$(P \land (Q \lor A)) \to ((P \land Q) \lor (P \land A))$$

Proof.

1	$((P \lor Q) \land (P \lor A)) \to (P \lor (Q \land A))$	add sentence hilb37
2	$(P \to Q) \to (\neg Q \to \neg P)$	add sentence hilb7
3	$(P \to A) \to (\neg A \to \neg P)$	replace Q by A in 2
4	$(B \to A) \to (\neg A \to \neg B)$	replace P by B in 3
5	$(B \to (P \lor (Q \land A))) \to (\neg(P \lor (Q \land A)) \to \neg B)$	replace A by $P \lor (Q \land A)$ in 4

6	$(((P \lor Q) \land (P \lor A)) \to (P \lor (Q \land A))) \to (\neg(P \lor (Q \land A)) \to \neg((P \lor Q) \land (P \lor A)))$	replace B by $(P \lor Q) \land (P \lor A)$ in 5
7	$\neg(P \lor (Q \land A)) \to \neg((P \lor Q) \land (P \lor A))$	MP with 1, 6
8	$\neg(P \lor \neg\neg(Q \land A)) \to \neg((P \lor Q) \land (P \lor A))$	elementary equivalence in 7 at 5 of hilb5 with hilb5
9	$\neg(P \lor \neg\neg(Q \land A)) \to \neg(\neg\neg(P \lor Q) \land (P \lor A))$	elementary equivalence in 8 at 12 of hilb5 with hilb5
10	$\neg(P \lor \neg\neg(Q \land A)) \to \neg(\neg\neg(P \lor Q) \land \neg\neg(P \lor A))$	elementary equivalence in 9 at 17 of hilb5 with hilb5
11	$\neg(P \lor \neg\neg(Q \land B)) \to \neg(\neg\neg(P \lor Q) \land \neg\neg(P \lor B))$	replace A by B in 10
12	$\neg(P \lor \neg\neg(C \land B)) \to \neg(\neg\neg(P \lor C) \land \neg\neg(P \lor B))$	replace Q by C in 11
13	$\neg(D \lor \neg\neg(C \land B)) \to \neg(\neg\neg(D \lor C) \land \neg\neg(D \lor B))$	replace P by D in 12
14	$\neg(D \lor \neg\neg(C \land \neg A)) \to \neg(\neg\neg(D \lor C) \land \neg\neg(D \lor \neg A))$	replace B by $\neg A$ in 13
15	$\neg(D \lor \neg\neg(\neg Q \land \neg A)) \to \neg(\neg\neg(D \lor \neg Q) \land \neg\neg(D \lor \neg A))$	replace C by $\neg Q$ in 14
16	$\neg(\neg P \lor \neg\neg(\neg Q \land \neg A)) \to \neg(\neg\neg(\neg P \lor \neg Q) \land \neg\neg(\neg P \lor \neg A))$	replace D by $\neg P$ in 15
17	$(P \land \neg(\neg Q \land \neg A)) \to \neg(\neg\neg(\neg P \lor \neg Q) \land \neg\neg(\neg P \lor \neg A))$	reverse abbreviation and in 16 at occurence 1
18	$(P \land (Q \lor A)) \to \neg(\neg\neg(\neg P \lor \neg Q) \land \neg\neg(\neg P \lor \neg A))$	elementary equivalence in 17 at 1 of hilb39 with hilb39
19	$(P \land (Q \lor A)) \to (\neg(\neg P \lor \neg Q) \lor \neg(\neg P \lor \neg A))$	elementary equivalence in 18 at 1 of hilb39 with hilb39
20	$(P \land (Q \lor A)) \to ((P \land Q) \lor \neg(\neg P \lor \neg A))$	reverse abbreviation and in 19 at occurence 1
21	$(P \land (Q \lor A)) \to ((P \land Q) \lor (P \land A))$	reverse abbreviation and in 20 at occurence 1

□

The second distributive law (second direction):

Theorem 0.6 (hilb41).

$$((P \land Q) \lor (P \land A)) \to (P \land (Q \lor A))$$

Proof.

1	$(P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))$	add sentence hilb36
2	$(P \to Q) \to (\neg Q \to \neg P)$	add sentence hilb7
3	$(P \to A) \to (\neg A \to \neg P)$	replace Q by A in 2
4	$(B \to A) \to (\neg A \to \neg B)$	replace P by B in 3
5	$(B \to ((P \lor Q) \land (P \lor A))) \to (\neg((P \lor Q) \land (P \lor A)) \to \neg B)$	replace A by $(P \lor Q) \land (P \lor A)$ in 4
6	$((P \lor (Q \land A)) \to ((P \lor Q) \land (P \lor A))) \to (\neg((P \lor Q) \land (P \lor A)) \to \neg(P \lor (Q \land A)))$	replace B by $P \lor (Q \land A)$ in 5
7	$\neg((P \lor Q) \land (P \lor A)) \to \neg(P \lor (Q \land A))$	MP with 1, 6
8	$\neg((P \lor Q) \land (P \lor A)) \to \neg(P \lor \neg\neg(Q \land A))$	elementary equivalence in 7 at 13 of hilb5 with hilb5
9	$\neg(\neg\neg(P \lor Q) \land (P \lor A)) \to \neg(P \lor \neg\neg(Q \land A))$	elementary equivalence in 8 at 4 of hilb5 with hilb5
10	$\neg(\neg\neg(P \lor Q) \land \neg\neg(P \lor A)) \to \neg(P \lor \neg\neg(Q \land A))$	elementary equivalence in 9 at 9 of hilb5 with hilb5
11	$\neg(\neg\neg(P \lor Q) \land \neg\neg(P \lor B)) \to \neg(P \lor \neg\neg(Q \land B))$	replace A by B in 10

12	$\neg(\neg\neg(P \lor C) \land \neg\neg(P \lor B)) \rightarrow \neg(P \lor \neg\neg(C \land B))$	replace Q by C in 11
13	$\neg(\neg\neg(D \lor C) \land \neg\neg(D \lor B)) \rightarrow \neg(D \lor \neg\neg(C \land B))$	replace P by D in 12
14	$\neg(\neg\neg(D \lor C) \land \neg\neg(D \lor \neg A)) \rightarrow \neg(D \lor \neg\neg(C \land \neg A))$	replace B by $\neg A$ in 13
15	$\neg(\neg\neg(D \lor \neg Q) \land \neg\neg(D \lor \neg A)) \rightarrow \neg(D \lor \neg\neg(\neg Q \land \neg A))$	replace C by $\neg Q$ in 14
16	$\neg(\neg\neg(\neg P \lor \neg Q) \land \neg\neg(\neg P \lor \neg A)) \rightarrow \neg(\neg P \lor \neg\neg(\neg Q \land \neg A))$	replace D by $\neg P$ in 15
17	$\neg(\neg(P \land Q) \land \neg\neg(\neg P \lor \neg A)) \rightarrow \neg(\neg P \lor \neg\neg(\neg Q \land \neg A))$	reverse abbreviation and in 16 at occurence 1
18	$\neg(\neg(P \land Q) \land \neg(P \land A)) \rightarrow \neg(\neg P \lor \neg\neg(\neg Q \land \neg A))$	reverse abbreviation and in 17 at occurence 1
19	$\neg(\neg(P \land Q) \land \neg(P \land A)) \rightarrow (P \land \neg(\neg Q \land \neg A))$	reverse abbreviation and in 18 at occurence 1
20	$((P \land Q) \lor (P \land A)) \rightarrow (P \land \neg(\neg Q \land \neg A))$	elementary equivalence in 19 at 1 of hilb39 with hilb39
21	$((P \land Q) \lor (P \land A)) \rightarrow (P \land (Q \lor A))$	elementary equivalence in 20 at 1 of hilb39 with hilb39

□

1 Cross Reference

This module is used by the following modules:

Name:	predtheo2
Version:	1.00.00
Rule version:	1.02.00
Orgin:	predtheo2_1.00.00_1.02.00.qedeq
pdf:	predtheo2_1.00.00_1.02.00.pdf